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Analysis of Stress by Integral Transforms Technique in Thermoelastic Hollow Cylinder

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ABSTRACT: The present paper deals with the determination of the unknown temperature, displacement and stress functions on the upper plane surface of a finite length hollow cylinder when the interior third kind boundary condition is known. Initially the plate is kept at zero temperature. A complete evaluation of temperature and stress distributions in a transient state is obtained using finite Marchi-Zgrablich and Laplace transform techniques. The results are obtained in series form in terms of Bessel's functions. The results for displacement and stresses have been computed numerically and illustrated graphically.

Key Words: Hollow cylinder, transient problem, thermoelastic problem, Marchi-Zgrablich-Laplace transform.

I. INTRODUCTION

The inverse thermoelastic problem consists of the determination of the temperature of the heating medium and the heat flux of a solid when the conditions of the displacement and stresses are known at some points of the solid under consideration. This inverse problem is very important in view of its relevance to various industrial machines subjected to heating such as main shaft of lathe and turbine and roll of a rolling mill.

In the works of Grysa and Cialkowski [1] and Grysa and Kozlowski [2], one dimensional transient thermoelastic problems are considered and the heating temperature and the heat flux on the surface of an isotropic infinite slab are derived. In the works of Khobragade and Wankhede [3], two dimensional steady-state thermoelastic problem is considered and the heating temperature, displacement and thermal stresses are derived. Singru [8] investigated thermal stress of a thick hollow cylinder. Evgeniy Dats [9] calculated the residual stresses of hollow cylinder under unsteady thermal action. Iryna Rakocha Popovych [10] developed the mathematical modeling and investigates the stress strain state of the three layer thermo sensitive hollow cylinder.

In the present problem an attempt is made to study the inverse transient thermoelastic problem to determine the unknown temperature, displacement and stress functions of the cylinder occupying the space

$$D:\{(x,y,z)\in R^3 : a \le (x^2+y^2)^{1/2} \le b, 0 \le z \le h\}$$

with the known interior third kind condition. The finite Marchi-Zgrablich and Laplace transform techniques are used to find the solution of the problem. Numerical estimate for the temperature distribution on the upper plane surface is obtained. A brief note contains relevant results of the transform, although elementary, are not easily found in textbooks provided in Appendix.

II. FORMULATION OF THE PROBLEM

Consider a hollow cylinder of length h occupying the space *D*. The differential equation governing the displacement function $\phi\ (r,z,t)$, where $r=(x^2+y^2)^{1/2}$ is

$$\nabla^2\phi = \frac{(1+\nu)}{(1-\nu)}\alpha_t\Theta \qquad ...(1)$$

with $\phi = 0$ at $r = a$ and $r = b$ ...(2)

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$

where ν and α_t are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the cylinder respectively and Θ is the temperature of the cylinder satisfying the differential equation

$$\frac{\partial^2\Theta}{\partial r^2}+\frac{1}{r}\frac{\partial\Theta}{\partial r}+\frac{\partial^2\Theta}{\partial z^2}=\frac{1}{k}\frac{\partial\Theta}{\partial t} \quad ...(3)$$

where k is the thermal diffusivity of the material of the cylinder,

subject to the initial condition

$$\Theta = 0 \text{ for all } a \le r \le b, \text{ and } 0 \le z \le h, \quad ...(4)$$

the interior condition

$$\left[\Theta+\frac{\partial\Theta}{\partial z}\right]_{z=\xi}=\xi(r,t) \text{ for all } 0\le\xi\le h, \text{ and } t>0 \quad ...(5)$$

and the boundary conditions

$$\left[\Theta+\frac{\partial\Theta}{\partial z}\right]_{z=0}=u(r,t) \text{ for all } a\le r\le b, \text{ and } t>0 \quad ...(6)$$

$$[\Theta]_{z=h}=g(r,t) \text{ for all } a\le r\le b, \text{ and } t>0 \quad ...(7)$$

$$\left[\Theta+k_1\frac{\partial\Theta}{\partial r}\right]_{r=a}=F_1(z,t) \text{ for all } 0\le z\le h, \text{ and } t>0 \quad ...(8)$$

$$\left[\Theta+k_2\frac{\partial\Theta}{\partial r}\right]_{r=b}=F_2(z,t) \text{ for all } 0\le z\le h, \text{ and } t>0 \quad ...(9)$$

The functions $F_1(z,t)$ and $F_2(z,t)$ are known constants and they are set to be zero so as to obtain mathematical simplicities. The constants k_1 and k_2 are the radiation constants on the two curved surfaces. The function $\xi(r,t)$ is assumed to be known while the function $g(r,t)$ is not.

The radial and axial displacement U and W satisfying the uncoupled thermoelastic equations [6] are

$$\nabla^2 U-\frac{U}{r^2}+(1-2\nu)^{-1}\frac{\partial e}{\partial r}=2\frac{(1+\nu)}{(1-2\nu)}\alpha_t\frac{\partial\Theta}{\partial r} \quad ...(10)$$

$$\nabla^2 W+(1+2\nu)^{-1}\frac{\partial e}{\partial z}=2\frac{(1+\nu)}{(1-2\nu)}\alpha_t\frac{\partial\Theta}{\partial z} \quad ...(11)$$

where $e=\frac{\partial U}{\partial r}+\frac{U}{r}+\frac{\partial W}{\partial z}$ is the volume dilation and

$$U=\frac{\partial\phi}{\partial r} \quad ...(12)$$

$$W=\frac{\partial\phi}{\partial z} \quad ...(13)$$

The stress functions are given by

$$\tau_{rz}(a,z,t)=0 \qquad \tau_{rz}(b,z,t)=0 \qquad \tau_{rz}(r,0,t)=0 \quad ...(14)$$

and

$$\sigma_r(a,z,t)=p_1, \qquad \sigma_r(b,z,t)=-p_0, \qquad \sigma_z(r,h,t)=0 \quad ...(15)$$

where p_1 and p_0 are the surface pressures assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components
by the following relations:

$$\sigma_r=(\lambda+2G)\frac{\partial U}{\partial r}+\lambda\left[\frac{U}{r}+\frac{\partial W}{\partial z}\right] \quad ...(16)$$

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left[\frac{\partial U}{\partial r} + \frac{U}{r}\right] \quad ...(17)$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left[\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z}\right] \quad ...(18)$$

$$\tau_{rz} = G\left[\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right] \quad ...(19)$$

where $\lambda = \frac{2G\nu}{1-2\nu}$ is the Lame's constant, G is the shear modulus and U and W are the displacement components. Equations (1) to (19) constitute the mathematical formulation of the problem under consideration [5, 6].

III. SOLUTION OF THE PROBLEM

The finite Marchi-Zgrablich integral transform of $\xi(r)$ is defined as

$$\bar{f}_p(m) = \int_a^b r\,\xi(r)\; S_p(\alpha,\beta,\mu_m\, r)\,dr \quad ...(20)$$

where α_1, α_2, β_1 and β_2 are the constants involved in the boundary conditions $\alpha_1\xi(r) + \alpha_2\;\xi'(r)\big|_{r=a} = 0$ and $\beta_1\xi(r) + \beta_2\;\xi'(r)\big|_{r=b} = 0$ for the differential equation $f''(r) + (1/r)f'(r) - (p^2/r^2)f(r) = 0$, $\bar{\xi}_p(n)$ is the transform of $\xi(r)$ with respect to kernel S_p and weight function r

The inversion of equation (20) is given by

$$f(r) = \sum_m \frac{1}{C_m}\,\bar{\xi}_p(m)\; S_p \quad ...(21)$$

where kernel function S_p can be defined as

$$S_p = J_p\,[G_p(\alpha,\mu_m a) + G_p(\beta,\mu_m b)] - G_p\,[J_p(\alpha,\mu_m a) + J_p(\beta,\mu_m b)] \quad ...(22)$$

being

$$J_p = J_p(\mu\varsigma) + \alpha\,\mu\, J_p'(\mu\varsigma) \quad \text{and} \quad G_p = G_p(\mu\varsigma) + \alpha\,\mu\, G_p'(\mu\varsigma)\ ,$$

where J_p and G_p are Bessel function of first and second kind respectively and

$$C_m = \int_a^b x\{S_p\}^2\,dx$$

$$= \frac{b^2}{2}\{S_p^2 - S_{p-1}(\alpha,\beta,\,\mu_m b)\cdot S_{p+1}(\alpha,\beta,\,\mu_m b)\} \; - \frac{a^2}{2}\{S_p^2 - S_{p-1}(\alpha,\beta,\,\mu_n a) S_p\}\,.$$

On applying finite Marchi-Zgrablich transform and Laplace transform to the equations (3) to (9) and then using their inversions, one obtain the expressions of temperature distribution and unknown temperature gradient respectively as

$$\Theta(r,z,t) = \frac{2k\pi}{\varsigma^2}\sum_{n=1}^{\infty}\frac{S_0}{c_n}\sum_{m=1}^{\infty}(-1)^m\, m[\theta\cos\theta\, z - \sin\theta\, z]$$

$$\times\int_0^t\left[\bar{\xi}(n,t') - \left[\frac{d\chi}{dz} + \chi\right]_{z=\xi}\right] e^{-k\left(\mu_n{}^2 + \frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}\, dt'$$

$$-\frac{2k\pi}{\varsigma^2}\sum_{n=1}^{\infty}\frac{S_0}{c_n}\sum_{m=1}^{\infty}(-1)^m m[\sin\theta(z-\varsigma)-\theta\cos\theta(z-\varsigma)]$$

$$\times\int_0^t\left[\overline{u}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=0}\right]e^{-k\left(a_n{}^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'+\sum_{n=1}^{\infty}\frac{S_0}{c_n}L^{-1}[\chi]$$

...(23)

$$g(r,t)=\frac{2k\pi}{\varsigma^2}\sum_{n=1}^{\infty}\frac{S_0}{c_n}\sum_{m=1}^{\infty}(-1)^m m[\theta\cos\theta h-\sin\theta h]$$

$$\times\int_0^t\left[\overline{\xi}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=\varsigma}\right]e^{-k\left(\mu_n{}^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'$$

$$-\frac{2k\pi}{\varsigma^2}\sum_{n=1}^{\infty}\frac{S_0}{c_n}\sum_{m=1}^{\infty}(-1)^m m[\sin\theta(h-\varsigma)-\theta\cos\theta(h-\varsigma)]$$

$$\times\int_0^t\left[\overline{u}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=0}\right]e^{-k\left(a_n{}^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'+\sum_{n=1}^{\infty}\frac{S_0}{c_n}L^{-1}[\chi] \quad ...(24)$$

where n is Marchi-Zgrablich transform parameter, χ stands for particular integral and is given by

$$\chi=e^{\mu_n z}\int\left[e^{-2\mu_n z}\int F(z)e^{\mu_n z}\,dz\right]dz$$

where $F(z)=\frac{a}{k_1}S_0(k_1,k_2,\mu_n a)F_1(z)-\frac{b}{k_2}S_0(k_1,k_2,\mu_n b)F_2(z)$,

μ_n are the positive roots of the equation $S_0(k_1,k_2,\mu_n r)=0$,

$\overline{f}(n,t)=\int_a^b r\,\xi(r,t)\;S_0 dr$, $\overline{u}(n,t)=\int_a^b r\,\xi(r,t)\;S_0\,dr$,

S_0 is kernel of the transform,

$$C_n=\int_a^b r\{S_0(k_1,k_2,\mu_n r)\}^2\,dr$$

and $\theta=\frac{m\pi}{\varsigma}$

Equations (23) and (24) are the desired solutions of the given problem with $\beta_1=\beta_2=1$ and $\alpha_1=k_1$, $\alpha_2=k_2$.

IV. DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting the value of Θ *(r,z,t)* from (23) in equation (1), one obtains the thermoelastic displacement function $\phi(r,z,t)$ as

$$\phi(r,z,t)=\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{2}\frac{k\pi}{\varsigma^2}\sum_{n=1}^{\infty}\frac{r^2S_0}{c_n}\sum_{m=1}^{\infty}(-1)^m m[\sin\theta\cos\theta z=-\sin\theta z\;]$$

$$\times\int_0^t\left[\overline{\xi}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=\varsigma}\right]e^{-k\left(\mu_n{}^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'$$

$$-\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{2}\frac{k\pi}{\varsigma^2}\sum_{n=1}^{\infty}\frac{r^2 S_0}{c_n}\sum_{m=1}^{\infty}(-1)^m m[\sin\theta z(z-\varsigma)-\theta\cos\theta(z-\varsigma)]$$

$$\times\int_0^t\left[\bar{u}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=0}\right]e^{-k\left(a_n^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'$$

$$+\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{4}\sum_{n=1}^{\infty}\frac{r^2 S_0}{c_n}L^{-1}[\chi] \quad ...(25)$$

Using (25) in (12) and (13) one obtains the radial and axial displacement U and W as

$$U=\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{2}\frac{k\pi}{\xi^2}\sum_{n=1}^{\infty}\left[\frac{r^2\mu_n S_0'+2rS_0}{c_n}\right]\times\sum_{m=1}^{\infty}(-1)^m m[\theta\cos\theta z-\sin\theta z]$$

$$\times\int_0^t\left[\bar{f}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=\varsigma}\right]e^{-k\left(\mu_n^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'-\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{2}\frac{k\pi}{\varsigma^2}\sum_{n=1}^{\infty}\frac{r^2\mu_n S_0'+S_0}{c_n}$$

$$\times\sum_{m=1}^{\infty}(-1)^m m[\sin\theta(z-\varsigma)-\theta\cos\theta(z-\varsigma)]$$

$$\times\int_0^t\left[\bar{u}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=0}\right]e^{-k\left(a_n^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'$$

$$+\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{4}\sum_{n=1}^{\infty}\left[\frac{r^2 S_0'\mu_n+2rS_0}{c_n}\right]L^{-1}[\chi] \quad ...(26)$$

$$W=\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{2}\frac{k\pi^2}{\varsigma^3}\sum_{n=1}^{\infty}\frac{r^2 S_0}{c_n}\sum_{m=1}^{\infty}(-1)^{m+1}m^2[\theta\sin\theta z+\cos\theta z]$$

$$\times\int_0^t\left[\bar{\xi}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=\varsigma}\right]e^{-k\left(\mu_n^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'$$

$$-\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{2}\frac{k\pi^2}{\varsigma^3}\sum_{n=1}^{\infty}\frac{r^2 S_0}{c_n}\sum_{m=1}^{\infty}(-1)^m m^2[\cos\theta(z-\varsigma)+\theta\sin\theta(z-\varsigma)]$$

$$\times\int_0^t\left[\bar{u}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=0}\right]e^{-k\left(a_n^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'$$

$$+\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{4}\sum_{n=1}^{\infty}\frac{r^2 S_0}{c_n}\frac{d}{dz}\left[L^{-1}[\chi]\right] \quad ...(27)$$

V. DETERMINATION OF STRESS FUNCTIONS

Using (26) and (27) in (16) the radial stress function is obtained as

$$\sigma_r=(\lambda+2G)\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{2}\frac{k\pi}{\varsigma^2}\sum_{n=1}^{\infty}\left[\frac{r^2\mu_n^2 S_0''+2r\mu_n S_0'+2S_0}{c_n}\right]\times\sum_{m=1}^{\infty}(-1)^m m[\theta\cos\theta z-\sin\theta z]$$

$$\times\int_0^t\left[\bar{\xi}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=\varsigma}\right]e^{-k\left(\mu_n{}^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'$$

$$-(\lambda+2G)\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{2}\frac{k\pi}{\varsigma^2}\sum_{n=1}^{\infty}\left[\frac{r^2\mu_n{}^2S_0{}''+2r\mu_nS_0{}'+2S_0}{c_n}\right]$$

$$\times\sum_{m=1}^{\infty}(-1)^m m[\sin\theta(z-\varsigma)-\theta\cos\theta(z-\varsigma)]$$

$$\times\int_0^t\left[\bar{u}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=0}\right]e^{-k\left(a_n{}^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt' +(\lambda+2G)\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{4}$$

$$\times\sum_{n=1}^{\infty}\left[\frac{r^2\mu_n{}^2S_0{}''+2r\mu_nS_0{}'+2S_0}{c_n}\right]L^{-1}[\chi]$$

$$+\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{2}\frac{k\pi}{\varsigma^2}\sum_{n=1}^{\infty}\left[\frac{r\mu_nS_0{}'+2S_0}{c_n}\right]\times\sum_{m=1}^{\infty}(-1)^m m[\theta\cos\theta z-\sin\theta z]$$

$$\times\int_0^t\left[\bar{f}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=\varsigma}\right]e^{-k\left(\mu_n{}^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'$$

$$-\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{2r}\frac{k\pi}{\xi^2}\sum_{n=1}^{\infty}\frac{r^2\mu_nS_0'+S_0}{c_n}$$

$$\times\sum_{m=1}^{\infty}(-1)^m m[\sin\theta(z-\varsigma)-\theta\cos\theta(z-\varsigma)]$$

$$\times\int_0^t\left[\bar{u}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=0}\right]e^{-k\left(a_n{}^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'$$

$$+\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{4}\sum_{n=1}^{\infty}\left[\frac{r^2S_0{}'\mu_n+2rS_0}{c_n}\right]L^{-1}[\chi]$$

$$+\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{2}\frac{k\pi^2}{\xi^3}\sum_{n=1}^{\infty}\frac{r^2S_0}{c_n}\times\sum_{m=1}^{\infty}(-1)^{m+1}m^2\theta[\theta\cos\theta z-\sin\theta z]$$

$$\times\int_0^t\left[\bar{f}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=\varsigma}\right]e^{-k\left(\mu_n{}^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt' -\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{2}\frac{k\pi^2}{\varsigma^3}\sum_{n=1}^{\infty}\frac{r^2S_0}{c_n}$$

$$\times\sum_{m=1}^{\infty}(-1)^m m^2\theta[\theta\cos\theta(z-\varsigma)-\sin\theta(z-\varsigma)]$$

$$\times\int_0^t\left[\bar{u}(n,t')-\left[\frac{d\chi}{dz}+\chi\right]_{z=0}\right]e^{-k\left(a_n{}^2+\frac{m^2\pi^2}{\varsigma^2}\right)(t-t')}dt'$$

$$+\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{\alpha_t}{4}\sum_{n=1}^{\infty}\frac{r^2 S_0}{c_n}\frac{d^2}{dz^2}\left[L^{-1}[\chi]\right] \qquad ...(28)$$

VI. SPECIAL CASE AND NUMERICAL RESULTS

Set $\xi(r,t)=(1-e^{-t})\varsigma\delta(r)$, $u(r,t)=(1-e^{-t})h\delta(r)$, $F_1(z,t)=F_2(z,t)=0$, $\alpha=\dfrac{2k\pi}{\varsigma^2}$, r = 0.75, a = 0.5, b = 1, h = 1, ς = 0.75, t = 1 sec, $k_1=0.25$, $k_2=0.25$ and k =0.38 in the equation (24) to obtain

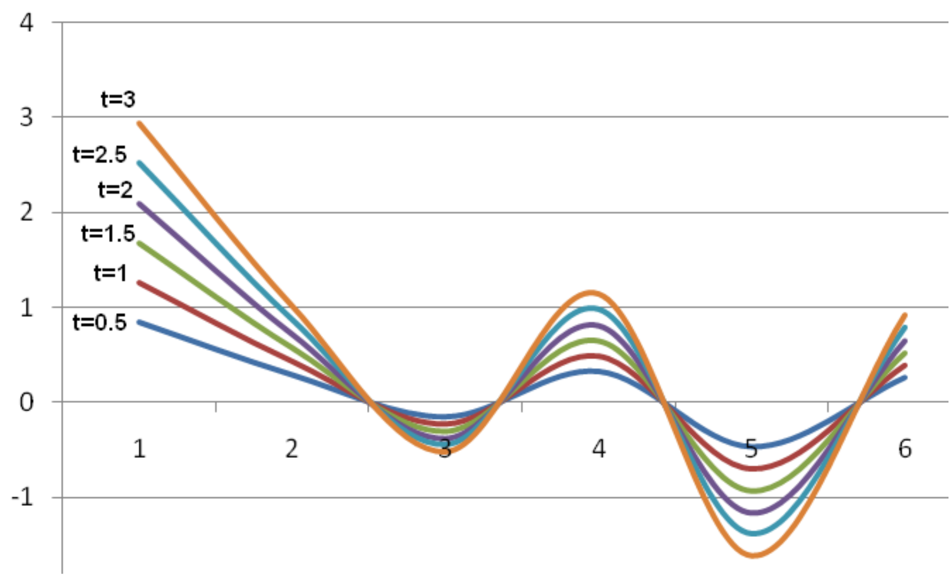
$$\begin{aligned}\frac{g(r,t)}{\alpha} &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{c_n}(-1)^m m[(4.25m)\cos(4.25m)-\sin(4.25m)]\\ &\times\left[\frac{1-e^{-0.38(\mu_n^2+17.3m^2)}}{\mu_n^2+17.3m^2}\right]\times(0.75)S_0^2(0.25,0.25,0.75\mu_n)\\ &-\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{c_n}(-1)^m m[\sin(10.5m)-(4.25m)\cos(10.5m)]\\ &\times\left[\frac{1-e^{-0.38(\mu_n^2+17.3m^2)}}{\mu_n^2+17.3m^2}\right]\times(0.75)S_0^2(0.25,0.25,0.75\mu_n)\end{aligned} \qquad ...(29)$$

VII. CONCLUSION

The temperature, displacement and stress functions at any point of the cylinder have been derived, when the interior third kind boundary condition and the other three boundary conditions are known, with the aid of finite Marchi-Zgrablich transform and Laplace transform techniques.
The expressions are obtained in the form of infinite series. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.
The results presented here will be more useful in Engineering problems particularly in the determination of the state of strain in the cylinder constituting the foundations of container for hot gases or liquid in foundations for furnaces etc.

VIII. GRAPHICAL ANALYSIS

Below figure shows the variation of radial stress versus z for different value of t.



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